

# The pricing of real options in discrete time models

## Another story of the value of waiting to invest

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**Keywords** *Real options, Modelling, Japan, Real estate*

**Abstract** *This paper proposes a discrete time real options model with time-dependent and serial correlated return process for a real estate development problem with waiting options. Based on a Martingale condition, the paper claims to be able to relax many unrealistic assumptions made in the typical real option pricing methodology. Our real option model is a new one without assuming the return process as "Ito Process", specifically, without assuming a geometric Brownian motion. We apply the model to the condominium market in Tokyo metropolitan area in the period 1971-1997 and estimate the value of waiting to invest in 1998-2007. The results partly provide realistic estimates of the parameters and show the applicability of our model.*

### 1. Introduction

Return process of a real estate does not follow independently and identically distributed process and not follow the "Ito Process". The price of real estate is not a random walk process, but has a serially correlated distribution which is not a normal distribution nor log-normal one. We develop a real-option pricing model without assuming the "Ito Process".

The traditional real-option pricing literature for real estate investments, such as Williams (1991), Quigg (1993), Capozza and Sick (1994), Grenadier (1995, 1996), etc. has been based on the problem of investment under uncertainty with the option to wait described in McDonald and Siegel (1986). We expand the traditional real-option pricing method by considering the time-dependency and serial correlation of real estate return process. McDonald and Siegel have discussed the investment problem to determine at what point in time, it is optimal to pay a sunk cost,  $I$ , when a project value evolves according to the following geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dz \quad (1.1)$$

where  $dz$  is the increment of a Wiener process.

They explore a rule that maximizes the value of investment opportunity, i.e. the value of the option to invest,  $W(V)$ .

$$W(V) = \max E[(V_t - I)\exp(-\rho t)] \quad (1.2)$$

subject to (1.1).  $s$  discount rate.

本稿は Journal of Property Investment & Finance, Vol.19, No.1, 2001, MCB University Press に掲載されたものである